Chapter 2

Harmony: The Traditional Debate Revisited

Logical inferentialism (sometimes called *proof-theoretic semantics*[1]) attempts to explicate the meaning of logical constants in terms of their use in deductive practice. Inferentialists typically distinguish between two types of use of a sentence that determine its meaning: the conditions for application of the concept (paradigmatically in assertion), and the consequences that follow from the application of such a concept. According to such a *two-aspect model* of language use, specifying the conditions for and consequences of the application of a given concept is sufficient to determine the meaning of such a concept. As Dummett (1973) puts it,

> "crudely expressed, there are always two aspects of the use of a given form of sentence: the conditions under which an utterance of that sentence is appropriate, which include, in the case of an assertoric sentence, what counts as an acceptable ground for asserting it; and the consequences of an utterance of it, which comprise both what the speaker commits himself to by the utterance and the appropriate response on the part of the hearer, including, in the case of assertion, what he is entitled to infer from it if he accepts it."(p. 396)

[1]This term was coined by Schroeder-Heister (1991). Strictly speaking, proof-theoretic semantics is broader than logical inferentialism due to its ambition to give a semantics for non-logical concepts (paradigmatically mathematical ones). Nonetheless, they operate on the same meaning-theoretic assumptions and goals.

Interpreting the two-aspect model in the natural deduction setting, the information specifying the conditions for application of a given logical constant is encoded in its introduction rule (I-rule). The I-rules specify the circumstances under which it is correct to introduce into the discourse (or proof) the complex formula whose principal operator is the logical constant in question. The consequences of application, on the other hand, include everything that is derivable from a given formula. In particular, the elimination rule(s) (E-rule) of a logical constant specify what we can correctly derive from a formula where such a constant figures in as the principal operator. Thus, introduction rules and elimination rules together completely determine the meaning of a logical constant. For example, the meaning of the constant $\rightarrow$ is understood to be captured by the following rules:

$$\frac{\begin{array}{c}[A]\\ B\end{array}}{A \rightarrow B}\ {\scriptstyle \rightarrow\text{-I}} \qquad \frac{A \qquad A \rightarrow B}{B}\ {\scriptstyle \rightarrow\text{-E}}$$

One of the first major objection to the logical inferentialist project is due to Prior (1960), where he (implicitly) gave a *reductio* argument against logical inferentialism by inventing the logical connective `tonk` governed by the following pair of operational rules:

$$\frac{A}{A \ \mathtt{tonk}\ B}\ {\scriptstyle \mathtt{tonk}\text{-I}} \qquad \frac{A \ \mathtt{tonk}\ B}{B}\ {\scriptstyle \mathtt{tonk}\text{-E}}$$

Assuming that transitivity holds—as it should—in the proof system to which it is added, `tonk` allows us to derive any arbitrary *B* for any arbitrary *A*. In other words, the addition of `tonk` to any proof system renders it deductively trivial. Of course, friends and foes of inferentialism alike agree that `tonk` is not a legitimate logical operator. But, Prior seems to argue, if meaning is wholly determined by its operational rules, why shouldn't `tonk`-like connectives count as legitimate and meaningful? From this, Prior's concludes that the inferentialist meaning-theoretic enterprise is untenable.

The objection amounts to the following: inferentialists are necessarily committed to conventionalism about logical laws and thus relativism about logical normativity. The usual inferentialist response to 'tonkitis' is to insist that the pair of operational rules that govern `tonk` are defective. Such introduction and elimination rules, according to inferentialists, are unjustified because they are not in *harmony* (a term coined by Dummett (1991)). Now, how exactly to spell out the idea that some rules are 'defective' or 'inharmonious' is still widely discussed. Because inferentialists are divided on the philosophical motivation for the requirement for harmony, we find in the literature various ways to formally cash out this idea.

This chapter contains three sections. In the first, I review the original articulation of the requirement of harmony as a local property of the pair of inference rules due to Gentzen (1969 [1935]) and developed in the proof-theoretic tradition by Prawitz (1965) and others. In the second section, I consider harmony as a global constraint on the deductive system as a whole; this view is originally due to Belnap (1962), but has enjoyed influential support from Dummett (1991). Finally, in section 3, philosophical motivations for the principle of harmony are considered. Despite the efforts of logical inferentialists, I argue that there is not a clear positive reason to adopt it.

2.1 Harmony as a Local Property

Prior's challenge primarily concerns inferentialists that subscribe to the two-aspect model of use, as the `tonk` connective exploits the possibility of the 'mismatch' between the introduction and elimination rules to render the proof system deductively trivial. The original inferentialist position, however, is immune to Prior's objection as it subscribes to a *one-sided model*, under which the two kinds of use (viz. conditions for concept application and the consequences for doing so) do not contribute equally to the meaning: the introduction rules are understood to fix the meaning and thus to determine the corresponding elimination rules. In the paper that launched the logical inferentialist tradition, Gentzen (1935; English translation in 1969) proposed that the meaning of logical constants is captured by rules that introduce them into a proof. According to him,

> "To every logical symbol $\&, \vee, \forall, \exists, \rightarrow, \neg$, belongs precisely one inference figure which 'introduces' the symbol—as the terminal symbol of a formula—and which 'eliminates' it. [...] The introductions represent, as it were, the "definitions" of the symbols concerned, and the eliminations are no more, in the

final analysis, than the consequences of these definitions. This fact may be expressed as follows: In eliminating a symbol, the formula, whose terminal symbol we are dealing with, may be used only "in the sense afforded it by the introduction of that symbol." [...] By making these ideas more precise it should be possible to display the E-inferences as single valued functions of their corresponding I-inferences, on the basis of certain requirements." (Genzten, 1969, p. 294-295)

An example might help illustrate what Gentzen has in mind. Take the pair of operational rules for the conditional described in the previous section. According to $\rightarrow$-I, we are permitted to endorse (in judgment or assertion) A $\rightarrow$ B only if we can derive B from assuming A. Thus, what the conditional makes explicit in the judgment is this inference and its hypothetical nature. What $\rightarrow$-E (also known as *modus ponens*) prescribes is that once there is a deduction of A, together with the fact that we can infer B from assuming A, we can directly endorse B. As such, elimination rules' sole function is to make explicit what we are entitled to in the first place when we introduced $\rightarrow$ in accordance with its meaning-determining introduction rule.

According to Gentzen's one-aspect model, then, i) introduction rules are definitional and thus self-justifying and ii) elimination rules are justified only if its application yield judgments whose grounds are already contained within grounds for applying the introduction rules.

2.1.1 Inversion Principle and Intrinsic Harmony

Prawitz (1965) develops Gentzen's programmatic remarks in his landmark study on natural deduction. He proposes that the relationship between the I- and E-rules can be captured in what he, following Lorenzen (1955), calls the *inversion principle*:

"Let α be an application of an elimination rule that has B as consequence. Then, deductions that satisfy the sufficient conditions [...] for deriving the major premises of α (if any), already "contain" a deduction of B; the deduction of B is thus obtainable directly from the given deductions without the addition of α." (Prawitz, 1965, p.33)

This principle is closely related to Gentzen's suggestion that the sole function of elimination rules is to 'undo' the inference made in accordance with the corresponding introduction rule. Importantly, the inversion principle guarantees that no novel information is produced when applying the elimination rules. For instance, lets take modus ponens ($\rightarrow$-E). The inversion principle tells us that the conditions for inferring A$\rightarrow$B (i.e. the conditions for applying $\rightarrow$-I, which is a deduction of B from [A]) and a deduction of minor premise A simply constitutes the deduction of B. That is, once a deduction of B from [A] and a deduction of A is available, we can directly infer B without applying $\rightarrow$-I and subsequently $\rightarrow$-E. This is because when the application of an introduction rule serves as a major premise of an elimination rule (which is the case for modus ponens), the deduction takes a detour which can be removed. Once all such detours are removed from the deduction, it is said to be *normalized* (or in the normal, or canonical, form).

The formal specification of this principle takes the form of the following *inversion theorem*:

> "If $\Gamma \vdash A$ then there is a deduction of A from Γ in which no formula occurrence is both the consequence of an application of an I-rule and major premise of an application of an E-rule." (Prawitz, 1965, p.34)

Let us call any formulae that occurs as the consequence of an application of an I-rule immediately followed by the application of an E-rule the *maximal formulae*, as it is the most logically complex formula compared to others in the surrounding deductive path. Prawitz then goes on to prove through induction that this inversion theorem holds for major deductive systems, including classical, intuitionistic, second-order, and modal logics.[2] In particular, Prawitz shows that for any (non-defective) pair of rules for a connective, there exists a procedure through which the deduction containing a maximal formulae

[2]The inversion theorem needs a slight addition to deal with the complication that arises with the $\perp$ rule for classical logic. While this fact may have bearing on the debate between classical and intuitionistic logicians, it is inconsequential for our purposes. Cf. Chapter 11 and 12 in Dummett (1991) for an attack on classical logic based on similar considerations.

can be reduced to one that does not contain one. Dummett refers to these reduction procedures as "levelling local peaks" (1991, p. 250) and operational rules that allow us to level local peaks as being in *intrinsic* harmony. Intrinsic harmony is a property belonging to the pair of rules governing the connective, and thus is *local*. Later, this local notion of harmony will be contrasted with a *global* notion of harmony which will be explained as the property of the deductive system as a whole. But first, let's take the case of the conditional to see how the leveling procedure works:

$$
\begin{array}{c}
\dfrac{\begin{matrix}\Pi_1\\ A\end{matrix} \qquad \dfrac{\begin{matrix}[A]\\ \Pi_2\\ B\end{matrix}}{A \to B}\,{\to}\text{-I}}{B}\,{\to}\text{-E}\\
\Pi_3
\end{array}
\qquad
\begin{matrix}\Pi_1\\ [A]\\ \Pi_2\\ B\\ \Pi_3\end{matrix}
$$

The deduction on the left side contains a maximal formulae, namely $A \to B$, but this peak can be leveled by arranging the demonstration of A (i.e. Π_1) and of $[A] \vdash B$ (i.e. Π_2) in a more linear fashion, as shown on the right side. Since there is a leveling procedure for the pair of rules governing $\to$, we have shown that they exhibit local harmony.

In like manner, Prawitz repeats this process with the four other connectives of first-order predicate logic. This inductive step of the proof of the inversion theorem, which utilizes leveling procedures, is integral to the main result of Prawitz's study, known as the *normalization theorem*: that every derivation in a system of natural deduction can be transformed into a normal form.[3] This result is, in essence, the natural deduction equivalent to Gentzen's *Hauptstaz* (also known as the *cut-elimination theorem*) for systems of sequent calculus. Indeed, as Prawitz and Gentzen (1938) himself notes, the Hauptstaz is also based on the insights articulated in the inversion principle.

But where does the requirement for harmony, conceived of as an existence of a reduction procedure, fit within the inferentialist meaning-theory? According to Martin-Löf (1996), "the place where [reduction procedure] belongs in the meaning theory is precisely in the semantical explanation, or justification, of the elimination rule" (p.33). Since, ac-

[3]It should be noted that leveling procedures, while integral to the proof of the normalization theorem, do not on their own establish it since the reduction of non-normal derivation to its normal form sometimes involve rearranging the deductive path to create a maximal formula in case where the succession of introductions and eliminations is not immediate. This procedure is known as *permutation*.

cording to the Gentzen-style one-sided model, introduction rules are self-justifying, the existence of a reduction procedure completes the justification of the pair of rules: expressions governed by intrinsically pair of rules can thus be considered fully justified.

To conclude this section, let us see how intrinsic harmony can help us with Prior's challenge. It should be clear that the `tonk` connective does not have a leveling procedure. In particular, Prior's runabout inference (i.e. the 'deduction' of an arbitrary B from an arbitrary A via the application of `tonk`-I and `tonk`-E) has a maximal formulae, namely $A\mathtt{tonk}B$, that occurs both as the consequence of application of an I-rule and as the major premise of an application of an E-rule; but, this deduction cannot be reduced to a normal form that avoids the detour since no such deduction exists. The problem with `tonk` can be thus understood as the following: its elimination rule outstrips (i.e. asserts more than what is granted by) the grounds for applying its introduction rule. As such, the rules that give meaning to the `tonk` constant do not exhibit local harmony. The explanation for this behavior is that the condition for applying `tonk`-I (namely, the proof of A) does not contain the grounds to judge B. So, the proof-theoretic diagnosis of the problem with `tonk` comes down to the following: the elimination rule for `tonk` is, given its introduction rule, unjustified.

2.1.2 Stability

Having shown tentatively that intrinsic harmony blocks `tonk`, we may now ask broader questions about Gentzen-style one-sided semantic model. Firstly, one might wonder why introduction rules are taken to be semantically primitive, as opposed to the elimination rule. In other words, why shouldn't elimination rules fix the meaning? Indeed, some have suggested that in the case of the conditional (Rumfitt, 2000, p.790), the universal quantifier (Dummett, 1991, p.275) and the disjunction operator (Tennant, 1987, p.90), it would be in some sense more natural to grant elimination rules definitional priority. Those working to vindicate proof-theoretic semantics often follow Gentzen in adopting the priority of the introduction rules over elimination rules, but seem to take this idea for granted. The choice to grant the introduction rules a kind of semantic priority requires justification, which is surprisingly missing from the vast literature on proof-theoretic semantics. We will provide a natural explanation for the priority of the introduction rules in the following chapter, drawing on the fact that we (i.e. rational agents) are embodied creatures that draw inference in time and are thus subject to temporal constraints.

Furthermore, granting that intrinsic harmony is a necessary property for any purportedly meaning-conferring pair of operational rules, we might still ask whether it on its own is constitute the sufficient condition. In other words, can intrinsic harmony rule out all `tonk`-like constants? In an influential argument, Dummett shows that intrinsic harmony is not sufficient in guaranteeing that the semantic properties of a constant is well-behaved. In particular, he concocts a counterexample by considering the case of the quantum logical disjunction.

The quantum-logical disjunction ($\ddot{U}$) differs from ordinary disjunction ($\vee$) in that its elimination rule does not allow for collateral hypotheses to be part of the minor premises. Since this is the sole difference, Prawitz's demonstration that a reduction procedure exists for $\vee$—and is therefore intrinsically harmonious—can be modified slightly to show that $\ddot{U}$ also is intrinsically harmonious. Dummett then imagines a system, S, containing $\ddot{U}$ and other intrinsically harmonious constants, with the notable exception $\vee$. Let's say that we want to create an extension of this system, S', by adding $\vee$ to it. All of the constants that of this system is intrinsically harmonious. Yet, as Dummett notes, something goes awry. In S', by exploiting $\vee$-I and $\ddot{U}$-E, we can derive $A \vee B$ whenever we have $A\ddot{U}B$, thus collapsing quantum disjunction into ordinary disjunction:

$$\dfrac{A\ddot{U}B \qquad \dfrac{[A]}{A \vee B}\,\vee\text{-I} \qquad \dfrac{[B]}{A \vee B}\,\vee\text{-I}}{A \vee B}\,\ddot{U}\text{-E}$$

In other words, in S', the conditions for asserting $A\ddot{U}B$ is sufficient to assert $A \vee B$. What exactly has gone wrong here? In short, the problem is that the elimination rule for $\ddot{U}$ is excessively restrictive in comparison to its introduction rule; the conditions for asserting $A\ddot{U}B$ is stronger than what the consequence of applying $\ddot{U}$-E makes explicit. So, like `tonk`, there is a mismatch between the introduction and the elimination rule, only the mismatch is of opposite nature. Whereas `tonk`-E is exceedingly permissive with respect to `tonk`-I, $\ddot{U}$-E is exceedingly restrictive with respect to $\ddot{U}$-I.

So far, the diagnosis has lead to an identification of a local problem, i.e. for the pair of operational rules for $\ddot{U}$. But the addition of $\vee$ to S creates problems at the *global* level as well, i.e. for the deductive system as a whole. For one, S' validates formula that are invalid in S; most notably, while the law of distributivity—$A \wedge (B\ddot{U}C) \iff (A \wedge B)\ddot{U}(A \wedge C)$—is invalid in S, it becomes derivable for $\ddot{U}$ in S'.[4] S' can thus be described as a *non-conservative extension* of S. Conservative extendibility is a key notion that serves as the basis for the requirement for harmony for Dummett and Belnap (1962); more on this will be said in the next section.

What's more, as Steinberger (2011) notes, a reduction procedure for a proof that contains both $\vee$ and $\ddot{U}$ does not exist in some cases. Let's say, for example, we want to extend the above proof of $A\ddot{U}B \vdash A \vee B$ by adding an application of $\vee$-E at the end.[5] In order to set up the reduction procedure by creating a maximal formulae, let us permutate the proof in the following way:

$$\dfrac{A\ddot{U}B \qquad \dfrac{\dfrac{[A]}{A \vee B}\,\vee\text{-I} \qquad \begin{matrix}\Gamma_1, [A] \\ C\end{matrix} \qquad \begin{matrix}\Gamma_2, [B] \\ C\end{matrix}}{C}\,\vee\text{-E} \qquad \dfrac{\dfrac{[B]}{A \vee B}\,\vee\text{-I} \qquad \begin{matrix}\Gamma_3, [A] \\ C\end{matrix} \qquad \begin{matrix}\Gamma_4, [B] \\ C\end{matrix}}{C}\,\vee\text{-E}}{C}\,\ddot{U}\text{-E}$$

Notice that in such a configuration, the application of $\ddot{U}$-E is not, in general, acceptable; it is only acceptable when the contexts (Γ_{1-4}) are all empty. What this shows is that such a proof cannot be reduced to a normal form, and consequently, that S' does not exhibit the global property of normalizability despite the fact that S does.

What the foregoing consideration—both at the local and global level—shows is that rules governing quantum-logical disjunction, despite them being intrinsically harmonious, do confer a *stable* meaning. For those who intend the requirement for harmony to be a cure for *tonkitis*, this should not be a worry as the requirement for intrinsic harmony takes care of Prior's counterexample. But for most logical inferentialists, the requirement for harmony is much more than that: they take it that a general account of harmony should specify the necessary and sufficient conditions for operational rules to genuinely confer meaning, thus assigning harmony a foundational role in the inferentialist meaning-theoretic project.

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[4]The failure of distributivity is in fact one of the key features of systems of quantum logic.

[5]This example is due to Steinberger (2011).

Of course, Dummett belongs to the latter camp of theorists, and thus concludes that intrinsic harmony is insufficient to qualify as such an account of harmony as such. He suggests that the requirement for intrinsic harmony should be complemented by counterpart, *stability*, which would rule out excessively restrictive elimination rules (Dummett, 1991, p.287). Although Dummett himself does not develop this notion very much, it has recently attracted the attention of many working to vindicate proof-theoretic semantics.[6] If intrinsic harmony is the requirement that a reduction procedure exists for any purportedly harmonious pair of rules, then stability may be formally cashed out as the requirement for an *expansion procedure*.[7] Take the conditional for example:

$$\begin{array}{c} \Pi \\ A \to B \end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\Pi \\ A \to B\end{array} \qquad [A]}{B}\,{\to}\text{-E}}{A \to B}\,{\to}\text{-I}$$

As shown, any proof Π of $A \to B$ (on the left side) can be expanded into a derivation that has an application of →-E followed subsequently by an application of →-I (on the right side). Since the pair of rules governing → has a expansion procedure, they can be said to exhibit stability.[8]

Interestingly, Dummett thinks that while the requirement for intrinsic harmony and stability[9] together constitute a sufficient condition for a general requirement of harmony, he claims that they are not necessary conditions. In other words, if all the constants in a given deductive system exhibit harmony and stability, then it suggests that we are in good shape; however, it is not strictly the *point* of harmony, according to Dummett. Instead, Dummett endorses the global (and relational) property of conservative extendibility as the most adequate general account of harmony. Dummett writes,

> "It is [harmony-as-conservative extendibility] that must prevail if the *point* of the requirement of harmony is to be attained, namely, that, for every logical constant, its addition to the fragment of the language containing only the other

[6]See, for example, Jacinto & Read (2017) and Tranchini (2018).

[7]Cf. Tranchini (2024) for details.

[8]The above expansion procedure in fact exists for the quantum logical disjunction; this

[9]These two aspects of harmony as a local property are aptly named by Schroeder-Heister (2014) as the 'criterion of reduction' and the 'criterion of recovery' respectively.

> logical constants should produce a conservative extension of that fragment" (Dummett, 1991, p.290, original italics).

Let us now turn to this idea.

2.2 Harmony as a Global Property

Belnap (1962) was the first to directly respond to 'tonkitis' on inferentialists' behalf. The main problem with admitting `tonk` to a deductive system, according to Belnap, is that it perturbs our current deductive practice and its central principles to which we are committed. He writes,

> "We are not defining our connectives *ab initio*, but rather in terms of an antecedently given context of deducibility, concerning which we have some definite notions. By that I mean that before arriving at the problem of characterizing connectives, we have already made some assumptions about the nature of deducibility." (1962, p.131)

Now, how exactly to spell out the "antecedently given context of deducibility" is a delicate issue that requires substantial discussion. For now, we shall content ourselves with the identification of such a context with a particular deductive system. A deductive system is defined by a set of rules, both operational and structural, and axioms that permit us to draw inferences. Concretely, what the admission of `tonk` undermines is the commitment to transitivity, which holds in any proof system worthy of the name.[10] Transitivity may remain implicit in the set of operational rules that constitute the deductive system (as is the case in natural deduction) or may be stated explicitly as a structural rule (as is the case in sequent calculus). Belnap adopts the latter approach, opting to explicitly codify transitivity in the form of a structural rule known as *cut*:

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,\text{Cut}$$

[10] Some recent developments in substructural logics (for example, Weir (2013)) challenge the idea that transitivity is indispensable to a deductive system, but I won't get into this issue here.

Given this antecedent commitment to transitivity, we can begin to formulate some criteria for admitting constants (and their corresponding rules) to the deductive system. In particular, in order to rule out constants governed by defective operational rules from ever entering our deductive system, Belnap proposes two criteria to determine the admissibility of logical constants to a given system. Of the two criteria for admissibility proposed, *conservativeness* is the key to ruling out `tonk`-like connectives.[11] Conservativeness amounts to the proviso that admitting a set of rules governing logical constants must not result in the derivability of new sequents (or, in Belnap's terminology, 'deducibility-statements') involving only those constants drawn from the system to which it is added. In other words, newly derivable sequents must involve the novel constant. It should be noted that, unlike intrinsic harmony and stability, conservative extendibility is a global property, i.e. a property of the entire deductive system. Furthermore, it is a relational property that holds between the base system and its extension.

Now, the trouble with `tonk`, according to Belnap, is that adding its operational rules to a deductive system yields a non-conservative extension of the previous system. By translating the operational rules for `tonk` as sequents and by exploiting the cut rule, we can show that, for any arbitrary A and B, B is derivable from A:

$$\frac{\Gamma, A \vdash A\texttt{tonk}B, \Delta \qquad \Gamma', A\texttt{tonk}B \vdash B, \Delta'}{\Gamma, A, \Gamma' \vdash \Delta, B, \Delta'}\ \text{Cut}$$

Given that the derived sequent—namely, $\Gamma, A \vdash \Delta, B$—was not already valid in the system prior to the addition of `tonk`, introducing it to a system yields a non-conservative extension. So, based on the conservativeness criterion, Belnap suggests that tonkish rules should not be admitted to our deductive system.

2.2.1 Separability and Atomism

Having shown that the requirement that the addition of a novel constant ought to yield a conservative extension of the base system safeguards against novel tonikish constants,

[11] The other criterion, *uniqueness*, guarantees the economy of the language. It can also be understood in some sense as a dual of conservativeness (cf. Dosen & Schroeder-Heister (1985)). Indeed, uniqueness can be seen as a global counterpart of stability, as it blocks the addition of a pair of rules in which elimination rules are too restrictive in comparison to the introduction rules.

we may now ask whether conservative extendibility is adequate as a general account of harmony that can serve as the foundation for the inferentialist meaning-theoretic project.

The most obvious point of departure is Dummett's (1991) influential discussion, since he understand a the requirement of harmony as "a precondition for the possibility of a compositional meaning-theory" (p.247), and, as noted earlier, thinks that such a requirement should be identified with conservative extendibility: "the point of the requirement of harmony is [...] that, for every logical constant, its addition to the fragment of the language containing only the other logical constants should produce a conservative extension of that fragment" (p.290). In other words, Dummett thinks that for a language (or a fragment thereof) to be good working order, the rules governing the inferential role of expressions must satisfy the conservative extendibility requirement in relation to expressions comprising the rest of the language.

Indeed, it is far from obvious why Dummett favors the identification of harmony with conservative extendibility, or more generally, harmony as a property of the entire deductive system, rather than as a local property of the pair of operational rules (viz. intrinsic harmony and/or stability). The general motivating thought seems to be the idea that the meaning of logical constants are determined completely by the pair of operational rules that specify its inferential role. In other words, the presence or absence of any other constant in the deductive system should not affect its inferential role in anyway. From this it follows that the addition of a novel constant should not validate formulas that are composed entirely of the logical vocabulary of the base system. As such, to uphold that the meaning of any logical constant is exclusively determined by its rules, Dummett, following Belnap, thinks that the conservativeness constraint ought to be in place.

The idea that the meaning of each logical constant should be exhaustively determined by its operational rules naturally leads to the requirement that deductive systems ought to be *separable*, i.e. that every derivable formula in the system has a proof that only involves either operational rules for the constant that figure in that formula, or structural rules. Furthermore, separability, combined with the usual inferentialist thesis that to understand the meaning of an expression is to know the all of the roles it can play when drawing inferences (see, for instance, P. A. Boghossian (2003)), entails inferential atomism: the idea that a speaker should be able to understand the meaning of a particular expression (say, $\wedge$), without understanding other expressions (say, $\vee$) of the same language.

In fact, many logical inferentialists subscribe to atomism. Tennant, for example, writes, "it should not matter in what order one learns [...] the logical operators" (Tennant, 1997, p.315). Dummett, too, shares this sentiment, at least when it comes to logical vocabulary. He writes, "to understand $A \vee B$, one need not understand $A \wedge B$ or $A \rightarrow B$" (Dummett, 1991, p.223). So, Dummett's identification of harmony with conservative extendibility can be seen as a way to guarantee the separability of logical constants, and, as a result, vindicating the inferentialist claim that the meaning of a logical constant is determined solely by its operational rules. The requirement for conservative extendibility, then, could be understood as the foundation of an inferentialist meaning-theory.

But, separability and the atomic conception of inferentialism—which, as we saw, go hand in hand—are controversial assumptions. As Milne (2002) argues, there are no positive arguments for the principle of separability beyond the assertion of an initally plausible but vague inferentialist credo. Furthermore, when inferentialist semantics is generalized to beyond logic to natural language, the principle of separability begins to appear implausible: why should one expect that we can learn the meaning of 'north,' for example, independently of learning the meaning of 'south'?[12] This latter consideration does not directly affect the cogency of the principle of separability when applied to deductive systems; however, logical inferentialists then owe us an explanation of i) why the semantics of logical constants should be treated in a privileged manner with respect to that of the rest of language and ii) what the criterion of logicality is, if we are to isolate logical constants and their meaning explanations from its natural language counterpart.

Dummett seems to be aware of these issues. This is why, as Steinberger (2011) notes, Dummett appears conflicted on whether or not to fully embrace the principle of separability: on the one hand, Dummett singles out logical constants "as a uniquely simple case" (Dummett, 1991, p.223) of expressions that can be learned in isolation from other expressions, on the other hand Dummett rejects that all introduction rules must be be

[12]Cf. Gareth Evans' discussion of the *generality constraint* for an argument supporting this idea.

‘pure’ like the Gentzen-Prawitz model we saw in the previous section. Commenting on the Gentzen-Prawitz requirement that each operational rule must mention only one logical constant (so as to guarantee separability), Dummett writes,

> “Reflection shows that this demand is exorbitant. An impure c-introduction rule [where c is some logical operator] will make the understanding of c depend on the prior understanding of the other logical constants figuring in the rule. Certainly we do not want such a relation of dependence to be cyclic; but there would be nothing in principle objectionable if we could so order the logical constants that the understanding of each depended only on the understanding of those preceding it in the ordering” (Dummett, 1991, p.257).

If Dummett’s claim that harmony should be understood as conservative extendibility rests on the principle of separability, then it appears to rest on shaky grounds, even for Dummett’s own standards. But the problem for harmony-as-conservative extendibility does not end there.

2.2.2 Is Deducibility Enough?

Earlier we bracketed the question of how exactly to spell out Belnap’s ‘the antecendently given context of deducibility’, and this is a good time to revisit it. Deducibility is by definition relative to each deductive system, so harmony as conservative extendibility, if it were to serve as the justification for operational rules, trades on the earlier question of what deductive system we are committed to in the first place. As Belnap notes,

> “It is good to keep in mind that the question of the existence of a connective having such and such properties is relative to our characterization of deducibility. If we had initially allowed $A \vdash B$ (!), there would have been no objection to tonk, since the extension would then have been conservative. Also, there would have been no inconsistency had we omitted from our characterization of deducibility the rule of transitivity.” (1962, p.133)

In a similar vein, as Ripley (2015) notes, once we give up the unrestricted use of the cut rule entirely, the addition of `tonk` to a system of sequent calculus yields a conservative extension, and thus, by Belnap's criteria, counts as a legitimate and meaningful connective. So, the question is, what determines—or better yet, *justifies*—the choice of a deductive system? Belnap seems to be relatively unbothered by this question, as he simply assumes that the context he has given (which includes the axiom $A \vdash A$ and four structural rules, viz. weakening, permutation, contraction, and transitivity) would be unassailed:

> "In accordance with the opinions of experts (or even perhaps on more substantial grounds) we may take this little system as expressing all and only the universally valid statements and rules expressible in the given notation: it completely determines the context." (1962, p. 132)

Belnap suggests that perhaps the justification of his deductive system would either consist of justification by the relevant authority or 'perhaps on more substantial grounds'. The first option is unacceptable for two reasons. First, in general, appeal to authority is a weak form of justification because it simply pushes the burden of justification, i.e. by giving a valid *epistemic reason* pertinent for a given claim, onto the authority in question. In the absence of a valid epistemic reason, justification granted by the authority amounts to nothing more than a disgruntled parent's declaration 'Because I said so!', accompanied by the illocutionary force of a command. Such an act lands us in the territory of dogmatism.[13] Secondly, justification by appeal to authority is problematic in this particular case because there is no consensus among the experts as to which deductive system expresses 'all and only the universally valid statements and rules', as the lively debate on logical pluralism and substructural logic attest to.[14] The problem for Belnapians is reformulated as the following: Can we still salvage harmony as conservative extendibility without the presupposition that there is a uniquely privileged deductive system?

The short answer is no. To be sure, there is no problem with the idea that a deductive system (call it S') containing a new operator is a conservative extension of the base system

[13]It might happen to be justified by a valid moral or pragmatic reason—e.g. 'clean your room, it's filthy!'—but is nonetheless epistemically unjustified.

[14]For example, I have mentioned in a previous footnote some logicians who work on intransitive logic, i.e. those that deny the structural rule of transitivity.

S given that $\Gamma \vdash_{S'} A$ only if $\Gamma \vdash_S A$, and to that extent S' displays harmony *relative* to S. But the problem with this construal of harmony is its focus on deducibility, which makes it an inherently relative notion. In the absence of the antecedent commitment to a particular deductive system, harmony-as-conservative extendibility could only serve as *conditional grounds* for the inferentialist meaning-theoretic project, i.e. conditional on the endorsement of a deductive system or another as the 'one true logic'. In other words, the justification for harmony-as-conservative extendibility rests on a prior justification for and endorsement of a particular deductive system. If the requirement for harmony is to serve as the foundation for a meaning-theory, such an approach seems too far-fetched if possible in principle.[15]

Now, contrast this conclusion with that of the previous section: "Since, according to the Gentzen-style one-sided model, introduction rules are self-justifying, the existence of a reduction procedure completes the justification of the pair of rules: expressions governed by intrinsically pair of rules can thus be considered fully justified." Therefore, intrinsic harmony (and stability) appear to be more promising than conservative extendibility as far as formulations of the requirement of harmony go.

2.2.3 Lorenzen on Admissibility

Before moving on from the topic of harmony-as-conservative extendibility, we should note that Belnap's criterion of conservativeness is historically preceded by Lorenzen (1955) who proposed similar restrictions to the *admissibility* of rules to a formal system (§2, 1955):

> "A rule R is called admissible in a calculus K, if its addition to the primitive rules of K—resulting in an extended calculus K + R—does not enlarge the set of derivable sentences. If $\vdash$ A denotes the derivability of A in K, then R is admissible if $\vdash_{\text{K+R}}$ A implies $\vdash_{\text{K}}$ A for every sentence A." (Schroeder-Heister, 2008, p.218)[16]

[15]In fact, I think that any theory of meaning that tacitly relies on the knowledge that our current inferential practice is correct is doomed from the get-go. I will comment more on this when we arrive at what the requirement for harmony amounts to when applied to the semantics of natural language.

[16]Much of the subsequent exegesis of Lorenzen (1955) is also due to Schroeder-Heister (2008).

Some clarification about Lorenzen's project and terminology is in order. The main goal of Lorenzen's *Operative Logic* (1955) is to justify intuitionistic logic based on principles of admissibility. The idea basic idea is not dissimilar from Belnap's: to begin with admissibility criteria, which are then applied to determine which rules of inference (in particular, operational rules for logical constants) are justified. However, Lorenzen's approach is more systematic and more general, since, unlike Belnap, Lorenzen does not start out by assuming an 'up-and-running' deductive context but instead begins with a general theory of formal systems (or what Lorenzen calls *Protologic*) in order to flesh out principles of admissibility; what Lorenzen refers to as 'calculus' in the above quote is equivalent to Curry (1951)'s notion of formal system, which include basic expressions and rules for producing complex expressions out of basic ones. Indeed, it would not be a stretch to say that Lorenzen was already trying to articulate, in the form of principles of admissibility, a theory of harmony understood in the Dummettian sense as the "precondition for a compositional meaning-theory," some 40 years before the term was even coined.

In Lorenzen's (1955) conception of logic, the notion of admissibility plays a fundamental role. In the above quote, implication is explained in terms of admissibility. Now, it is tempting to explain admissibility in terms of implication,[17] but obviously, such a strategy would yield viciously circularity. So, Lorenzen explains admissibility by refering to the notion of an *elimination procedure* (Lorenzen, 1955, §3). As Schröder-Heister sums up,

> "R is admissible in K, if every application of R can be eliminated from every derivation in K + R. The implicational relation between existential statements expressed in [the statement $\vdash_{K+R}$ A implies $\vdash_K$ A] is reduced to the insight that a certain procedure reduces any given derivation in K + R in such a way that the resulting derivation does no longer use R" (Schroeder-Heister, 2008, p.218).

So, admissibility is analyzed as the existence of a kind of a procedure—a kind of 'doing' (i.e. a concrete task) rather than a kind of 'saying' (i.e. an assertion), to quote Brandom's oft-cited turn of phrase.[18] The specification of this procedure is the content of,

[17] Along the following lines: $\vdash$ A implies $\vdash_R$ A only if R is admissible.
[18] Cf. R. Brandom (2008).

in Lorenzen's terminology, the 'operative meaning' of admissibility. In this sense, Lorenzen's notion of admissibility shares the same pragmatist (i.e., use-theoretic) underpinning as intrinsic harmony, which was analyzed earlier in the chapter as the existence of a procedure (namely, to reduce any derivation into its normal or canonical form).[19] By analyzing admissibility in terms of the elimination procedure, Lorenzen understands implication to signify more than derivability, namely, as an expression of the *understanding* gained through the elimination procedure which underpins the notion of admissibility.

Let us now compare Lorenzen's notion of admissibility with Belnap's conservativeness constraint. *Prima facie*, admissibility is just the conservativeness constraint without the proviso Belnap adds in order to allow for complex statements involving novel constants. But this is too quick. While Lorenzen insists on a distinction between the notion of admissibility and derivability,[20] Belnap's sole focus is on derivability. Admissibility, as we saw, has an epistemic component: the operative meaning of admissibility is based on the ability to perform the elimination procedures, which yields further understanding through such performances and at the same time demands the appropriate kind of know-how to do so. On the other hand, derivability is a formal notion that simply describes the syntactic relationship between assumptions and theorems. Belnap's failure to appreciate the epistemic significance of the activity of reasoning, in my view, is part of the reason why his criteria of conservative extendibility, which describes the relationship between systems with differing 'deducibility-statements', is irreducibly relativistic and thus fails to *justify* the logical inferentialist project in any normatively significant way.[21]

To repeat my diagnosis of the problem with harmony-as-conservative extendibility, conservative extendibility has no all-things-considered normative significance when taken as a purely syntactic constraint. Lorenzen's approach to admissibility is illuminating in

[19] As was noted earlier, Lorenzen was also the first to articulate the 'inversion principle'—from which intrinsic harmony springs—as one of the principles of admissibility.

[20] As Schröder-Heister (2008) notes, "the standard proof-theoretic example of a rule admissible but not derivable is that of the cut rule in the first-order sequent calculus" (p.220).

[21] Indeed, it is the conceptual distinction between admissibility and derivability is what prompts Schröder-Heister to write, "it is the admission of this sort of evidence which makes Lorenzen an intuitionist rather than a formalist" (ibid.). Here, 'intuitionist' is understood roughly as taking 'construction,' an epistemic achievement of the reasoner, to be fundamental to logic (and mathematics) while 'formalist' is understood roughly to take structures describable in purely syntactic terms to be fundamental to logic.

that it allows us to conceive of the constraint we set for meaning-determining rules in a explicitly normative terms since justification is built into the very notion of admissibility. Having said that, Lorenzen's (1955) project of *Operative Logic* failed with respect to capturing the meaning of implication (namely, the fact that the assumption can be discharged when introducing the connective without possessing the proof of it). This led Lorenzen to the what is known as his 'dialogical turn,' wherein features related to meaning (of constants) was treated on one level (the 'play level') and features related to validity was treated on another level (the 'strategy level'). For now, I will merely flag this issue only returning to it in the final chapter.

2.3 What is the Point of Harmony?

So far, I have introduced the requirement for harmony as a inferentialist response *contra* Prior's `tonk` challenge. I have shown that, despite their differences, both the local account of harmony (i.e. the relationship between each introduction and elimination rules must exhibit intrinsic harmony and stability) and the global account (i.e. the deductive system as a whole must exhibit properties like conservative extendibility) are formally adequate in safeguarding against tonkish constants. Tonkers' reply would naturally be to question the legitimacy of harmony. Imposing such a requirement on every set of rules governing connectives, the thought goes, seems to run contrary to the oft-cited inferentialist slogan of *meaning as use*. By all means, it is true that the `tonk` connective is not intrinsically harmonious (locally) and its addition to first-order predicate calculus creates a host of problems at the global level: it creates a non-conservative extension of the system, alters the inferential roles (and thus the meaning) of other constants, and renders the system non-normalizable. But if the meaning of logical constants are solely determined by the rules governing them, why shouldn't `tonk` count as perfectly meaningful if undesirable or unjustified? That is, the claim that `tonk` is defective because its addition to standard deductive systems render them useless (a practical consideration), the claim that rules governing `tonk` are unjustified (an epistemic consideration), and the claim that rules governing `tonk` do not really confer meaning (a semantic consideration) appear to be three distinct claims.

Here, the inferentialist has two realistic options.[22] They might concede that tonkish constants are perfectly meaningful if its pair of rules unjustified; or, they might press on with the stronger claim, namely, that tonkish constants are not meaningful at all. In either case, we need an explanation of why the requirement of harmony could serve as the justification of logical laws or as the criterion for meaningfulness of logical constants. Since the epistemic and semantic significance of the requirement of harmony is not often kept apart, we find much confusion in the literature. So, it would be prudent to take extra care to keep them apart:

- **Epistemic interpretation of harmony**: Rules that fail to satisfy the requirement of harmony are unjustified.
- **Semantic interpretation of harmony**: Expressions governed by rules that fail to satisfy the requirement of harmony are incoherent or meaningless.

To be sure, both the epistemic interpretation and the semantic interpretation of the requirement of harmony is individually sufficient to refute Prior's initial argument. If rules governing `tonk` are defective, i.e. unjustified or incoherent, then Prior's initial objection to logical inferentialism loses its bite. But, in order to truly vindicate the inferentialist approach to meaning, we need to interpret harmony first and foremost as a semantic thesis rather than an epistemic one.

2.3.1 Principle of Innocence

While the literature on harmony is vast, much of the attention has been paid to technical details; those that explicitly deal with the motivation of the requirement harmony is scant. In a rare treatment of this issue, Steinberger (2009) makes the following plausible suggestion regarding the philosophical motivation for the requirement for harmony:

> **The principle of innocence**: logic alone should not be a source of new information. That is, it should not be possible, solely by engaging in deductive

[22] Here, I rule out the practical reply for the same reasons that I gave when criticizing Belnap's argument in section 2.3.

> reasoning, to discover hitherto unknown (atomic) truths about the world that we would have been incapable of discovering (at least in principle) independently of logic. (Steinberger, 2009, p.60)

The principle of innocence follows from an intuitive and commonly accepted understanding of what a deduction is. Unlike other forms of reasoning—for example, induction or abduction—deductive reasoning does not aim at uncovering novel information about the (empirical) world. Since the principle of innocence only motivates intrinsic harmony and conservative extendibility, given the preceding discussions, we may also add its counterpart to the effect that there is no information loss, in order to motivate the local property of stability (and the global property of uniqueness-preservation): it should not be possible, solely by engaging in deductive reasoning, to say less about the world than when we started off. Given this proviso, the principle of innocence seems to motivate all forms of harmony discussed so far. Thus, according to Steinberger, "The role of harmony [...] is precisely to protect the innocence of logic" (ibid.). Indeed, Dummett at times also seems to make this very point.[23] Dummett's identification of harmony with conservative extendibility could be thus be understood as resulting from his commitment to the principle of innocence-that the addition of logical laws should not validate hereto invalid formula and thus change the meanings of expressions already contained in the language.

Now, the principle of innocence implies that the logical fragment of language should be clearly demarcated from the rest of the language. For, if inferential relations that constitute the meaning of logical expressions and those that constitute the meaning of non-logical expressions were connected, of what should logic be innocent? This observation gives rise to the following thesis:

> **The principle of autonomy**: the logical fragment is self-contained. That is, the meaning of a logical constant cannot depend on the meanings of non-logical expressions. (Ibid.)

The principle of innocence and the principle of autonomy swim or sink together. Not only is the principle of autonomy implied by the principle of innocence, the converse also

[23]cf. for example Dummett, 1991, p.219

holds: without a clear demarcation, we begin to lose the boundary between expressions that are 'innocent' (i.e. whose meanings do not affect and are not affected by conceptual changes that alter the meaning of ordinary expressions) and those that aren't.

If, as Steinberger suggests, the principle of innocence motivates the requirement of harmony, then the requirement of harmony is best understood as a justification for laws of logic (i.e. inference rules for logical constants); in turn, the justification of the requirement of harmony rests on the justification of the principle of innocence and on the tenability of the principle of separability and of autonomy (which are a collateral commitments of the principle of innocence). In this way, logical laws are justified by appealing to our understanding of what logic is, or ought to be.

I will not deal here with this larger question about the nature of logic, in particular, whether or not the principle of innocence correctly captures an important feature of logic. For our purposes, it suffices to observe that whatever the 'correct' conception of logic is, appealing to the principle of innocence as the basis of the requirement of harmony has several important consequences for the larger inferentialist meaning-theoretic project. Firstly, it implies that the semantics of natural language can in principle proceed in isolation from logical inferentialism. So long as the principle of autonomy holds, there is no positive reason to explain the semantics of natural language in inferential terms. For example, a theorist could plausibly subscribe to truth-conditional semantics for natural language while explaining the meaning of logical constants in inferential terms.[24]

Relatedly, given the principle of innocence, the requirement of harmony now is most straightforwardly seen as a *criterion for logicality*. This amounts roughly to the claim that, in order for an expression to count as a logical constant, it must satisfy the requirement of harmony (however we formally cash it out). So, quasi-connectives like `tonk` are ruled out as *logical* constants. They are ruled out on the basis that its introduction and elimination rules, since they violate the requirement of harmony, are unfit to count as a logical constant; that is, given a prior understanding of what logic ought to be, `tonk`'s meaning-conferring rules are unjustified.

[24]That said, it should be noted that most likely no logical inferentialst would adopt this position. This mere fact already indicates that there is probably a reason more fundamental than the principle of innocence that motivates the requirement for harmony.

Crucially, the choice of motivating the requirement of harmony through the principle of innocence seems to commit us to the epistemic interpretation of harmony. Again, the foregoing discussion shows that the principle of innocence *justifies* the requirement of harmony, which in turn justifies the rules for (legitimate) logical constants. Indeed, there is no obvious way in which the principle of innocence might sustain the semantic interpretation (unless logicality is paraphrased as 'meaningfulness qua logical constant'). On this view, tonkish expressions seem to be perfectly meaningful (as the meaning is adequately specified by introduction and elimination rules) if the rules that give meaning are unjustified.

So far we have been exploring the explanation and some consequences of justifying requirement of harmony by appealing to the thought that logic should be innocent. This argument is intuitive enough, as it rests on the commonly accepted conception of what logic—in particular, deductive reasoning—ought to be. Unfortunately, this strategy is problematic, at least for two reasons. Firstly, as I have hinted at earlier, the fact that the motivation so conceived for the requirement of harmony is logic-specific (i.e. does not apply to the semantics of the rest of language) is a problem for the larger inferentialist project. Beyond the fact that the principle of innocence fits in an ad-hoc manner within the deeper pragmatist (viz. 'meaning as use') theoretical picture that underpins inferentialism, objections along Prior's lines are legion regarding inferentialism about natural language. Furthermore, if we think, as inferentialists typically do,[25] that it is instructive to "look at the contents of logical concepts as providing the key to conceptual content generally" (R. Brandom, 2007, p.161), the appeal to the principle of innocence as the basis for the requirement harmony—which holds the key to the vindication of logical inferentialism—dooms the project from the get-go.

The applicability of the requirement of harmony to natural language semantics is a substantial topic that requires careful attention; thus, I will dedicate the next chapter to this issue. For now, it is enough to note that even theorists like Dummett, who subscribes to the principle of innocence, think that rules governing any purportedly meaningful

[25] For example, R. Brandom (2000), Dummett (1991), and Peregrin (2014), amongst others.

expression, logical or otherwise, should be harmonious.[26] This suggests *prima facie* that there is a deeper motivation behind the requirement harmony beyond the principle of innocence.

The claim that the point of harmony is to secure the innocence of logic creates a rift between semantics of logical constants and of non-logical expressions, but of course, this rift on its own does not directly affect the cogency of the argument when presented in isolation from global inferentialist commitments. Nonetheless, the rift should be understood as a symptom of a underlying problem, namely that considerations from the principle of innocence fails to support the semantic interpretation of harmony. As it will be shown in the following discussion, the interpretation of harmony as an epistemic constraint is, while necessary, not sufficient for inferentialists; the interpretion of harmony as a semantic constraint—namely, that expressions governed by inharmonious rules are incoherent or meaningless—is crucial in order to vindicate the inferentialist meaning-theoretic project at-large against usual doubts put forth by its opponents.

2.3.2 "Syntax is insufficient for Semantics"

The philosophical intuition that underlies Prior's doubt runs deeper than what the `tonk` argument makes explicit. Following Searle (1980)'s (in)famous Chinese room argument, we might put it as a slogan: *Syntax is neither sufficient for nor constitutive of semantics*. Indeed, it is the very intuition shared by most contemporary theorists of semantics for both formal and natural languages that are skeptical about the inferentialist meaning-theoretic project. My contention is that the correct understanding of harmony—namely, the semantic interpretation—will help dispel this worry and vindicate the inferentialist approach.

According to Stevenson (1960), the moral that should be drawn from Prior's argument is that operational rules cannot define the meaning of a connective—which leaves room for `tonk`—since meaning, properly speaking, should be stated in terms of truth-functional statements in the metalanguage. According to this line of thinking, there is an

[26] We will discuss Dummett's view—that any meaningful expressions should exhibit harmony—at length in the following chapter.

antecedent notion of what, for example, "or" (or more precisely, statements of the form "A or B") means independently of the role it plays in inference; an effective way to capture this antecedent meaning is by specifying the truth conditions, either in the metalanguage or via truth tables. What the invention of `tonk` shows, then, is that in the absence of a truth-functional—and more generally, value-functional—explanation of meaning of connectives in the metalanguage, operational rules lose its semantic (and thus normative) significance since the semantic significance of rules come down to how well it behaves in accordance with the semantics we specify antecedently and independently of setting the rules.

There is a sense in which Stevenson's conclusion is trivially true. In modern logic after Tarski (1935), one tends to conflate the term 'semantics' with model theoretic semantics, according to which formal systems are first assumed to be uninterpreted symbols which needs to then be assigned a meaning by specifying the suitable function that maps uninterpreted symbols (in the object language) to its 'extension' (in the metalanguage). One of the assumptions of this project is that syntax—in the narrow sense as concerning rules determine the well-formedness of a given string of symbols—is largely independent of semantics so conceived.[27] This line of thought originates in the metamathematics tradition inaugurated by Hilbert, whose goal was to show that mathematical reasoning can be explicitly stated in a precise formal language. As Sundholm (1997) notes,

> "[...] so called metamathematical expressions, that is, inductively generated strings, or trees, of atomic metamathematical "signs", are not made to express anything, but on the contrary are themselves, like any other mathematical objects expressed by means of ordinary linguistic expressions. Metamathematical expressions are not tools for speaking with, but objects to be spoken about" (p.191-192).

[27] Largely but not entirely, since it is not difficult to see that the meaning of a complex formula is composed *syntactically* from atomic formulae, even if atomic formulae are themselves given a model-theoretic semantics. So, the very commitment to compositionality (which is usually a desiderata of any meaning-theory) suggests that syntactic features are at least part of the meaning of some expressions. Taking this idea further, Read (2008), drawing on examples from modal logic, observes that "[The demand for a truth-functional meaning explanation] is clearly unreasonable, as there are many non-truth-functional connectives, forming complex propositions whose truth-value is not determined by any finite set of "truth"-values of their constituents" (p.286).

Even after Gödel conclusively demonstrated that Hilbert's program (viz. formalizing all mathematical statements) cannot be sustained, its founding assumptions have lived on. Indeed, its effects can be felt throughout the history of modern logic, of which Stevenson's elaboration of Prior's argument is one. Sundholm writes, "It is fair to say, however, that without the Hilbert program, and the ensuing metamathematical codification by, say, Gödel, Tarski, and Bernays one would not have thought of studying formal languages consisting of uninterpreted strings of objects" (ibid., p.191). This metalogical view is, at this point, deeply entrenched in the philosophical imagination. Indeed, metalogical assumptions are usually embedded in critiques of logical inferentialism, old and new.[28]

Now, inferentialists characteristically resist the bifurcation of the analysis of language, both formal and natural, into syntax and semantics. If we abandon the idea that formal language—i.e. at the object language level—consists purely of meaningless symbols, then Stevenson's objection loses the bite. The entire inferentialist meaning-theoretic project rejects that an 'antecendent meaning,' namely the extensional specification of the referents, exists for logical connectives and atomic formulae alike. So, Stevenson's objection appears to be misguided. Nonetheless, Stevenson's exegesis of Prior's `tonk` argument is illuminating because it makes explicit the disagreement between tonkers and anti-tonkers which runs at this deeper level: the rub is created by what each camp understands to be the correct relationship between syntax and semantics. In order to dispel doubts about inferentialism without begging the question, we need to deal with this conceptually prior issue.

In the following sections, I aim to show that harmony—conceived as a criterion for meaningfulness—is fundamental to the inferentialist defense of syntax and semantic as irreducibly connected features with can be made explicit at the same level (viz. the object language).

[28]Compare, for example, Rumfitt (2017): "there is a far better explanation of why 'tonk' is meaningless than that it violates Harmony. It is not meaningful, because a formula whose main connective it is does not say anything; such a formula does not say anything because it does not have truth-conditions."

2.3.3 Alternatives to the Metalogical view

Inferentialists are united in rejecting the metalogical assumption, namely, that formal language consists of uninterpreted string of symbols whose meanings are specified in the metalanguage.[29] But what exactly is wrong with it? Consider the following critique of Tarski's semantic project due to F. B. Fitch (1944):

> "The metalanguage in which a general semantics would be formalized would not be of much use unless it could take itself as one of its object languages, for otherwise a general semantics could never deal in detail with a sufficiently large group of formal languages. It would always have to treat its own formalization as outside the scope of its pronouncements" (p.68).

Fitch's concern is that, from a practical, computational point of view, the metalanguage needs to be given a semantics by embedding itself to the object language. But the same point could be argued from the pragmatist philosophical point of view. The driving idea is that meaning cannot be settled from 'the view from nowhere,' i.e. outside (the object) language, whether natural or formal. And, if meaning is essentially use, then the project of semantics qua specifying meaning using language would never be complete, as practices change and evolve; this is another meaning of Brandom's turn of phrase, 'Semantics must answer to pragmatics.' Given this philosophical backbone, if Sundholm is correct to characterize metamathematical expressions not "as tools for speaking with, but objects to be spoken about," the metalogical assumption must give. (In fact, the reasons behind logical inferentialists' rejection of the metalogical view is similar to that of pragmatists' rejection of representationalism as presented in the previous chapter.)

The alternative to the metalogical view is to understand language as *always* interpreted. The most influential source of this alternate view can be found in Frege's early writings. In the *Begriffsschrift* ('Concept-script'), Frege tacitly assumes that any well-formed formula (of the object-language) are meaningful, i.e. interpreted. Objects that

[29] It should be noted that there are other criticisms to the Tarskian definition of truth (and of logical consequence as necessary truth-preservation given this notion of truth) from within the model-theoretic tradition, most notably Etchemendy (1990).

are derived in Frege's system are judgments (or assertions), which are contentful; judgments stand in contrast with well-formed formula of modern logic, which are syntactic objects that do not express content unless interpreted.[30] Because assertions are expressed in the object language (thanks to the use of the turnstile, which serves as the 'force indicator'), Frege's formalism allows one to make explicit inferential relations involving assertions (with propositional content) in addition to derivability relations involving (uninterpreted) well-formed formula.

Since assertion is an implicit knowledge claim (as explained in the previous chapter), the epistemological slant was part and parcel of Frege's early inferentialism. In §3 of the *Begriffsschrift*, he gives an inferential definition of propositional content. Frege analyzes the propositional content of a judgment A as what B has in common with any judgment that can be substituted in its place as a premise in valid arguments. In other words, A and B have the same propositional content just in case in any arbitrary context Δ and for every formula C,

$$\Delta, A : C \text{ is valid if and only if } \Delta, B : C \text{ is valid.}$$

Since Frege's work precedes the metamathematical turn in the early 20th century, he does not explicitly criticize a metalogical approach to meaning. The (rather underdeveloped) meaning explanation that Frege gives in §29, however, is clearly incompatible with the metalogical approach. For Frege, as Sundholm notes, "meaning explanation proper for the mathematical language is not itself mathematical in nature: one does not give a semantics for the language of mathematics by doing further mathematics, for example, by proving theorems" (1997, p.193).

Frege's early epistemological approach to logic and foundations of mathematics was then developed in the *constructivist* (or *intuitionistic*; I shall use the terms interchangably) tradition of Brouwer, Heyting, and Kolmogorov. Constructivists reversed the metalogical order of explanation: rather than beginning with a formal axiomatization then specifying a suitable semantics, which then is used to prove the soundness of that language, con-

[30] Sometimes it is suggested that Frege abandoned his earlier inferentialism and embraced a more metalogical approach to semantics after his *Sinn und Bedeutung*. This need not bog us down for the purposes of our discussion.

structivists took as explanatory primitives the notion of a *proposition*. Then, they defined how we can use them in our deductive practices, namely what it is for a proposition to be true, when someone is entitled to assert the proposition, what acts of inference are truth-preserving, etc.[31]

Most important for our discussion is that, in the constructivist tradition, proposition is analyzed as its proof-conditions (i.e. what one needs to do in order to produce an adequate demonstration), or in an alternative but essentially similar veil, as a problem to be solved.[32] Thus, it is fair to say that two theoretical orientations form the foundation of constructivist semantic theorizing: first, the metaphysically-tinted question, 'what is the meaning of x?' is replaced by the epistemologically-tinted question, 'what is it for a speaker to understand the meaning of x?'; second, propositional knowledge ("know-that") is explained in terms of ability ("know-how"). Inferentialists, including Brandom and especially Dummett, adopt both of these insights—i.e. the epistemological and pragmatist orientation to theorizing—from constructivism.

2.3.4 Semantic Significance of Harmony

In light of this diagnosis of the disagreement between *tonkers* and inferentialists, what can we say about the requirement of harmony? Let us return to Dummett's claim that harmony is "a precondition for the possibility of a compositional meaning-theory," for now we have the resources to appreciate this claim fully.

First, let's clarify why compositionality is a desideratum of a meaning theory. There is the empirical fact that all natural languages are *productive*, in the sense that no language has an upper bound for the number of well-formed, contentful sentences. The syntactic explanation for the productivity of language is Chomsky's celebrated notion of *recursion*; nonetheless, the fact that those sentence are meaningful (and thus interpreted) can only be explained by invoking the semantic notion of *compositionality*: roughly, that principle that the meaning of a complex expression is determined by the meanings of its constituents and the rules used to combine them. Recall that, to the question, 'What is it for a speaker

[31]See Sundholm (1983) for a clear and thorough presentation of this idea.
[32]The former equivalence is due to Heyting (1930) and the latter to Kolmogorov (1932).

to understand the meaning of an expression?', inferentialists would answer, 'To be able to use it in accordance with the correct rules of inference involving that expression.' In other words, it is the theoretical desideratum of a general meaning theory to articulate the what kind of epistemic conditions must be in place for speakers to engage in inferential practice using specific expressions. So, if we want to articulate a meaning theory of how speakers actually use language, compositionality must be accounted for.

But why is harmony strictly necessary for this phenomenon? Elaborating on Brouwer's criticism of classical mathematics, Dummett writes,

> "[Language] must stand up to the critical scrutiny of its functioning [and such scrutiny] aims at a systematic means of ascribing content to the expressions and sentences of the language, in terms of which accepted modes of operating with it (including the rules of inference observed) can be justified, or, better, are evidently justified. The mere existence of a common practice in using it, a practice that can be learned and that incorporates a means of distinguishing conventionally correct uses from incorrect ones, did not, for Brouwer, guarantee that that practice was coherent. *It is incoherent when there is disharmony between its component conventions, or equivalently, when it cannot be rendered comprehensible by any systematic ascription of content*—and this means, eventually, when *it would be impossible to construct a meaning-theory that accorded with all features of the practice*. We have seen that a practice may exist, and be learnable, but still open to criticism as failing to achieve the purpose that a linguistic practice should serve. That was precisely what Brouwer believed to be true of the practice of classical mathematics" (1991, p.240-241, my italics).

Why does Dummett think that disharmony between rules can create a situation where it is "impossible to construct a meaning-theory that accorded with all features of the practice?" This is left opaque in *The Logical Basis of Metaphysics*, but a brief thought experiment will illustrate the intution. Imagine a (formal or natural) language composed only of *tonkish* expressions—i.e., expressions whose condition for application and consequences of application are intrinsically inharmonious and unstable, and/or expressions whose addition to the base language creates a non-conservative extension. Even if the external observer (say, a radical interpreter in Davidson's sense) observes some regularities in patterns of behaviors of the 'speakers' of this language, there could not possibly be any property to properly call *meaning*, as the 'inferential' role of any expression could not possibly be learned, at least by any human, and probably by any rational being. Indeed, the relations between expressions can only be understood quasi-inferentially, in that some expression literally follows another. So, one can only concoct the `tonk` example against a background of a meaningful language, i.e. a sufficiently coherent set of rules and expressions that (mostly) respect the principle of compositionality.

So, on Dummett's count, harmony is first and foremost a semantic constraint. Expressions (or a group of them) governed by rules that fail to satisfy the requirement of harmony are incoherent or meaningless. Indeed, the fact that such rules give rise to incoherence is the basis on which Dummett advances the epistemic interpretation: Rules that fail to satisfy the requirement of harmony are unjustified.

Does Dummett's argument succeed? There are reasons to doubt it. Dummett rightly thinks that compositionality is a desideratum of any meaning theory. He also thinks, for reasons unrelated to the discussion on harmony, that the inferentialist approach is the correct way to theorize about meaning. Given these two assumptions, the principle of harmony must hold. But the inferentialists' opponent (like Prior) attack inferentialism on the basis that it leads to conventionalism and thus relativism about conceptual norms. In order to dispel the opponent's worries without begging the question, Dummett must provide a *positive* reason for why the principle of harmony should hold. Indeed, Dummett (1991) doesn't seem to provide any positive reason to regard harmony as something that should hold independently of our antecedent commitment to the inferentialist approach to meaning. That is, since inferentialist approach to semantics is correct and since compositionality must be accounted for, Dummett thinks harmony must hold if we are to 'harmonize' these two commitments. But even if Dummett is right to think broadly that inferentialism is correct, we can always put into doubt the cogency for the requirement for harmony (as, we will see in Chapter 3, Brandom does) within the larger inferentialist enterprise.

2.4 Conclusion

This chapter took a historical tour of the literature on the principle of harmony, which was formulated by a number of theorists in the wake of Prior's (1960) classic objection against logical inferentialism. In the discussion, two themes emerged: first, that harmony, if such a requirement could be vindicated, should take the form of a *local property* of the pair of rules governing a constant (*qua* intrinsic harmony and stability) over Belnap's and Dummett's *global* formulation (*qua* conservative extendibility). Secondly, we examined the philosophical motivations for the principle of harmony: in particular, we looked at the Principle of Innocence (as a justification for the epistemic interpretation of harmony) and Dummett's argument for the semantic interpretation (based on considerations from compositionality). In either case, neither of these strategies were found to be convincing.

The discussion of this chapter, in conjunction with Chapter 1 (on Brandom's inferentialism and it's account of objectivity), sets up the second half of this work. If the principle of harmony can be adequately grounded, then we secure one form of objectivity, understood as the *attitude-transcendence* of norms. Recall that a norm n is attitude-transcendent (AT) for community C iff it is possible for everyone in C to be mistaken about the correct applications of n. Even if a linguistic community has unanimous agreement about the correct applications of n, if n results in disharmony, then it is wrong to endorse n.[33] Given that the principle of harmony secure us AT, two questions arise: Could the principle of harmony apply to rules governing concepts of natural language, and what *grounds* such such a constraint? The first question is the topic of Chapter 3; the second question is the dealt with in the Chapter 4.

[33]Of course, the principle of harmony itself is a norm; thus, it cannot be precluded that, even if we endorse harmony, we are nonetheless collectively wrong to endorse this norm. In this sense, harmony does not secure us attitude-transcendence of norms. But this is a more general problem about philosophy (or theorizing more generally) and the possibility of objectivity. This will, hopefully, be a topic I will explore on another occasion.

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